When Primordial Black Holes from Sound Speed Resonance Meet a Stochastic Background of Gravitational Waves

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Content

- Sound speed resonance (SSR) mechanism
- The induced GWs in the SSR mechanism
- Summary

Sound Speed Resonance (SSR) Mechanism

SSR Mechanism

- A phenomenological model with a time-dependent c_s^2:

$$S_{\delta\phi} = \int d^3x dt a^3 \frac{1}{2}\left[\dot{\delta\phi}^2 - \frac{1}{a^2}c_s^2(\partial_i\delta\phi)^2\right]$$

- Oscillating sound speed:

$$c_s^2 = 1 - 2\xi[1 - \cos(2p_*\tau)] \text{ with } \tau > \tau_i$$

The amplitude: ξ is small and $\xi < 1/4$ such that c_s^2 is positively definite.
The characteristic scale: p_* is the oscillation frequency.
The beginning of oscillation: $|p_*\tau_i| \gg 1$.

- Equation of Motion:

$$v_p'' + \left(c_s^2 p^2 - \frac{z''}{z}\right) v_p = 0$$

Mukhanov-Sasaki variable: $v = z\zeta$ $\quad z = \sqrt{2\epsilon}a/c_s \text{ with } \epsilon \equiv -\dot{H}/H^2$

Y.-F. Cai, X. Tong, D.-G. Wang, and S.-F. Yan, Phys. Rev. Lett. 121, 081306 (2018)

SSR Mechanism

The EoM can be rewritten in the standard form of the Matheiu equation:

$$\frac{d^2 v_p}{dx^2} + \left(A_p - 2q\cos 2x\right) v_p = 0$$

where

$$x \equiv k\tau,\ \ A_p \equiv \frac{p^2}{p_*^2} + 2q - 4\xi,\ \ q \equiv \left(2 - \frac{p^2}{p_*^2}\right)\xi$$

Parametric resonance

- **Non-trivial mode function** (combinations of the Mathieu sine and Mathieu cosine functions):

$$v_{p_*}(\tau) = \frac{H\tau}{\sqrt{2p_*}} e^{iv} \frac{S(s)\left(iC(v) - C'(v)\right) + C(s)\left(-iS(v) + S'(v)\right)}{-S(v)C'(v) + C(v)S'(v)}$$

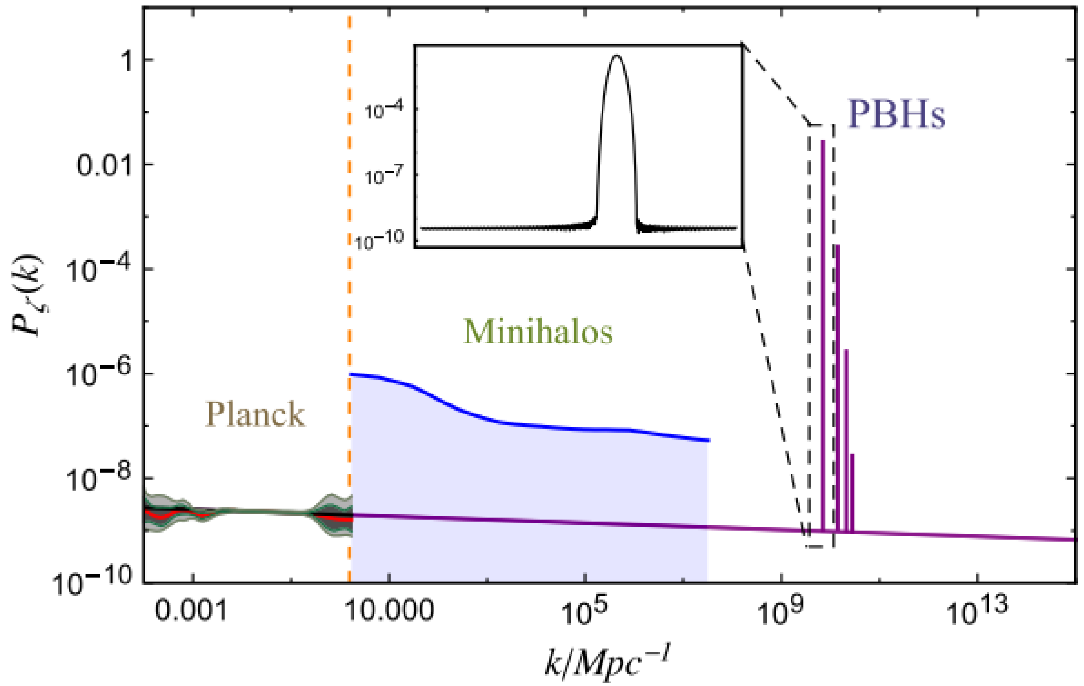
where

$$s \equiv -p_*\tau, v \equiv -p_*\tau_i, C(x) \equiv C(1,\xi,x),\ S(x) \equiv S(1,\xi,x),$$
$$C'(x) \equiv \partial C(1,\xi,x)/\partial x,\ S'(x) \equiv \partial S(1,\xi,x)/\partial x.$$

- **Enhanced curvature perturbation:**

$$P_\zeta(p) = A_s \left(\frac{p}{k_p}\right)^{n_s - 1} \left\{1 + \frac{\xi p_*}{2} e^{-\xi p_* \tau_i} \left[\delta(p - p_*) + \sum_{n=2}^{\infty} a_n \delta(p - n p_*)\right]\right\}$$

Enhancement factor

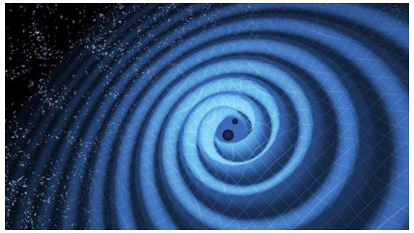


Spikes on small scales, scale-invariant on large scales

Induced Gravitational Waves in the SSR Mechanism

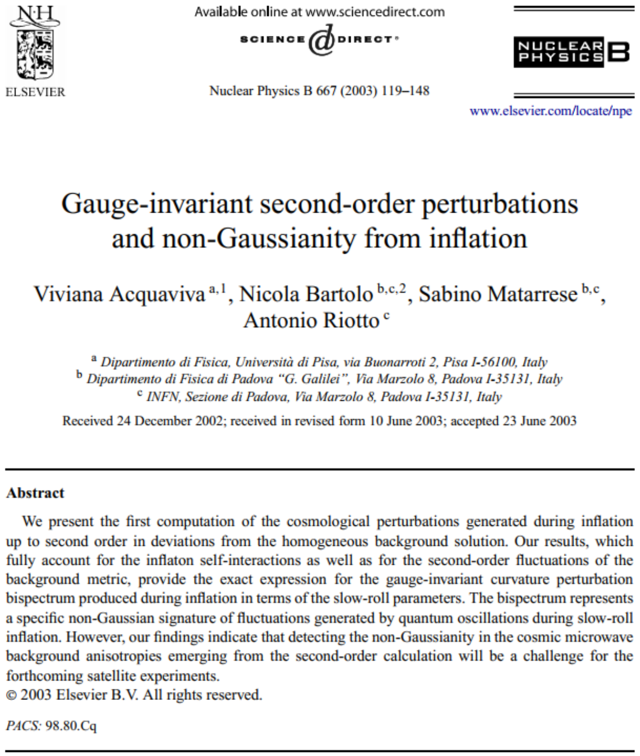
PBHs as the Sources of the Gravitational Waves

- **PBHs as the sources of GWs:**



  - PBHs merger:
    Merger events observed by LIGO (solar masses)

  - **Induced GWs:**
    At the non-linear level, different k-modes of scalar perturbations couple with each other and could be the sources of the tensor perturbations, i.e. the GWs.



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Gauge-invariant second-order perturbations and non-Gaussianity from inflation

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Abstract

We present the first computation of the cosmological perturbations generated during inflation up to second order in deviations from the homogeneous background solution. Our results, which fully account for the inflaton self-interactions as well as for the second-order fluctuations of the background metric, provide the exact expression for the gauge-invariant curvature perturbation bispectrum produced during inflation in terms of the slow-roll parameters. The bispectrum represents a specific non-Gaussian signature of fluctuations generated by quantum oscillations during slow-roll inflation. However, our findings indicate that detecting the non-Gaussianity in the cosmic microwave background anisotropies emerging from the second-order calculation will be a challenge for the forthcoming satellite experiments.

PACS: 98.80.Cq

Idea: The enhanced primordial density perturbation might induce the large GWs! These could be another indirect detect way of PBHs.

- **The Background:**

Spatially flat Friedman-Robertson-Walker (FRW);

Friedmann equations;

- **The EoM for the GWs:**

$$h''_{ij}(\tau,\mathbf{x}) + 2\mathcal{H}h'_{ij}(\tau,\mathbf{x}) - \nabla^2 h_{ij}(\tau,\mathbf{x}) = -4\hat{\mathcal{T}}_{ij}^{lm} S_{lm}(\tau,\mathbf{x})$$

TT part of the sources

TT projection:
$$\hat{\mathcal{T}}_{ij}^{lm}\mathcal{S}_{lm}(\tau,\mathbf{x}) = \sum_{\lambda=+,\times}\int \frac{\mathrm{d}^3\mathbf{k}}{(2\pi)^3} e^{i\mathbf{k}\cdot\mathbf{x}} e_{ij}^{\lambda}(\mathbf{k}) e_{lm}^{\lambda}(\mathbf{k}) \mathcal{S}_{lm}(\tau,\mathbf{k})$$

In Fourier space:
$$h_{\mathbf{k}}^{\lambda\,''}(\tau) + 2\mathcal{H}h_{\mathbf{k}}^{\lambda\,'}(\tau) + k^2 h_{\mathbf{k}}^{\lambda}(\tau) = S_{\mathbf{k}}^{\lambda}(\tau)$$

The two polarizations might not be equivalent in the present of source terms.

where
$$S_{\mathbf{k}}^{\lambda}(\tau) = -4e_{lm}^{\lambda}(\mathbf{k})\mathcal{S}_{lm}(\tau,\mathbf{k})$$

Basic Formulae for the Gravitational Waves

- **The solution of the GWs (Green function method):**

$$h_{\mathbf{k}}^{\lambda}(\tau) = \int_{-\infty}^{\infty} \mathrm{d}\tau_1 g_{\mathbf{k}}(\tau, \tau_1) S_{\mathbf{k}}^{\lambda}(\tau_1)$$

where $$g_{\mathbf{k}}''(\tau, \tau_1) + 2\mathcal{H} g_{\mathbf{k}}'(\tau, \tau_1) + k^2 g_{\mathbf{k}}(\tau, \tau_1) = \delta(\tau - \tau_1)$$

- **The power spectrum:**

$$\langle \hat{h}_{\mathbf{k}}^{\lambda}(\tau) \hat{h}_{\mathbf{k}'}^{s}(\tau) \rangle = (2\pi)^3 \delta^{\lambda s} \delta^{(3)}(\mathbf{k} + \mathbf{k}') \frac{2\pi^2}{k^3} \mathcal{P}_h(k, \tau)$$

The canonical quantization of the GWs (Ford&Parker1977):

$$\hat{h}_{\mathbf{k}}^{\lambda} = h_k^{\lambda}(\tau) \hat{a}_{\mathbf{k}}^{\lambda} + h_k^{\lambda *}(\tau) \hat{a}_{-\mathbf{k}}^{\lambda \dagger}$$

The equal time canonical commutation:

$$[\hat{a}_{\mathbf{k}}^{\lambda}, \hat{a}_{\mathbf{p}}^{s\dagger}] = \delta^{\lambda s} (2\pi)^3 \delta^{(3)}(\mathbf{k} - \mathbf{p}),$$

$$[\hat{a}_{\mathbf{k}}^{\lambda}, \hat{a}_{\mathbf{p}}^{s}] = [\hat{a}_{\mathbf{k}}^{\lambda\dagger}, \hat{a}_{\mathbf{p}}^{s\dagger}] = 0 \ .$$

Basic Formulae for the Gravitational Waves

- **The energy spectrum (a stochastic background of the GWs):**

$$\Omega_{\mathrm{GW}}(\tau,k)=\frac{1}{\rho_c(\tau)}\frac{\mathrm{d}\rho_{\mathrm{GW}}(\tau,k)}{\mathrm{d}\ln k}$$

The effective energy density of the GWs:

$$\rho_{\mathrm{GW}}=\frac{1}{32\pi G a^2(\tau)}\langle h'_{ij}h'_{ij}\rangle$$

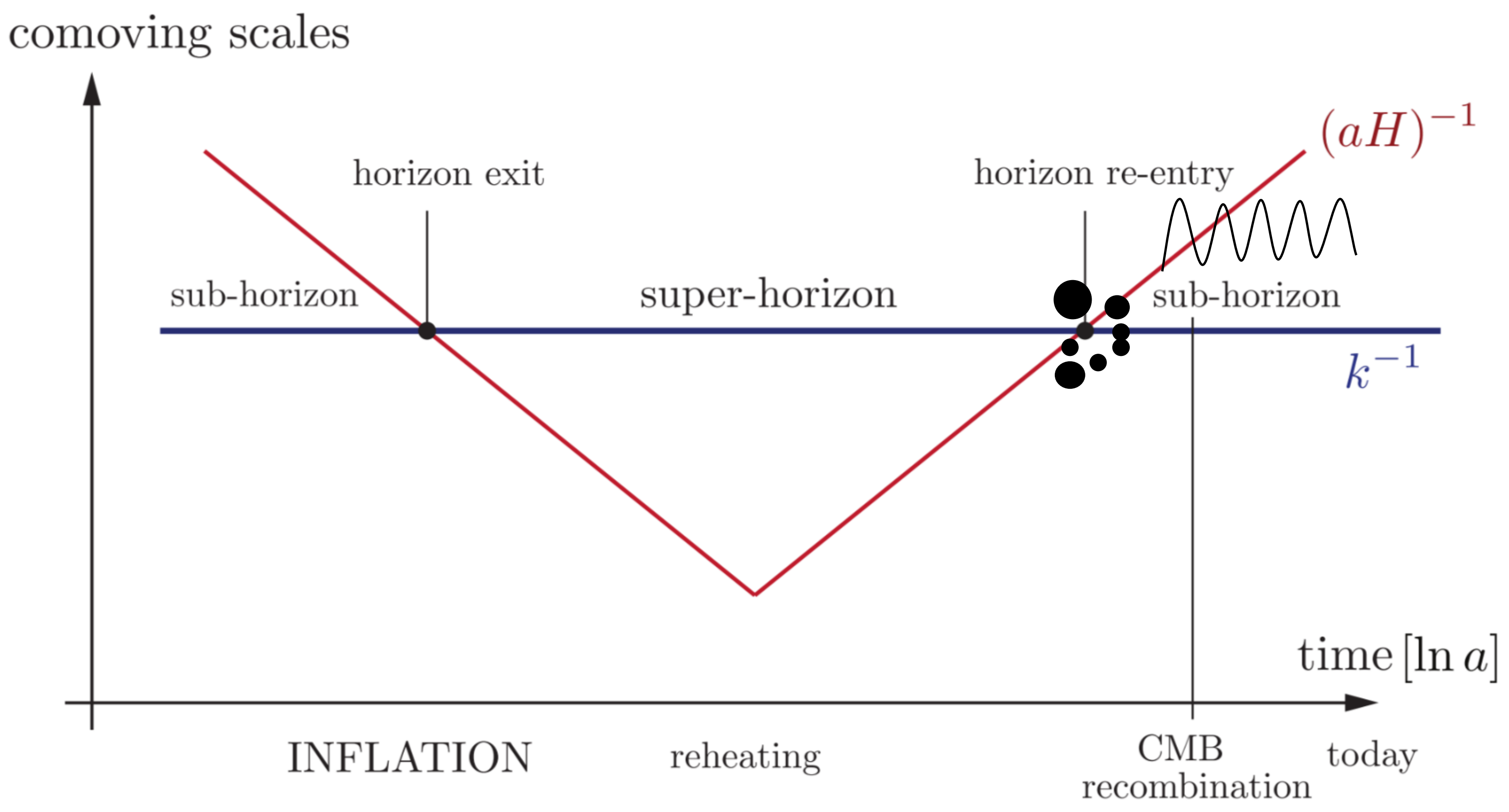
Many problems,
but most used one.

The relation to P_h:

$$\Omega_{\mathrm{GW}}(\tau,k)=\frac{1}{24}\left(\frac{k}{\mathcal{H}}\right)^2\overline{\mathcal{P}_h(k,\tau)}$$

Induced Gravitational Waves

➢ **The induced GWs associated with PBHs formation:**



Credited by D. Baumann

Induced Gravitational Waves

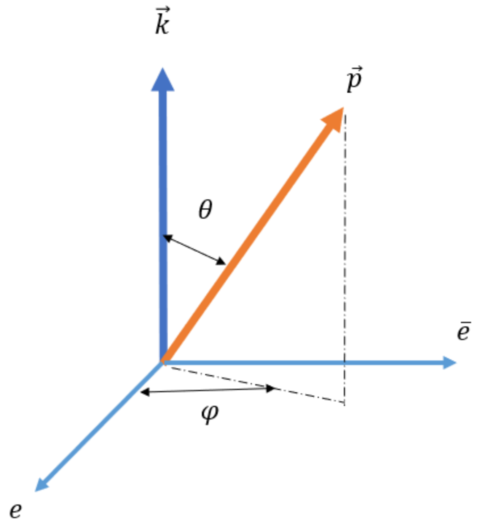
➢ **The perturbed spacetime in Newtonian gauge:**

$$d^2s = a^2(\tau)\Big\{-(1-2\Phi)d\tau^2+\Big[(1+2\Phi)\delta_{ij}+\frac{1}{2}h_{ij}\Big]dx^i dx^j\Big\}$$

where Φ is the Bardeen potential and h_{ij} is the 2nd-order tensor perturbation.

➢ **The source term during the radiation-dominated era:**

$$S_{\mathbf{k}}^{\lambda}(\tau) = 4\int \frac{\mathrm{d}^3\mathbf{p}}{(2\pi)^3}\mathbf{e}^{\lambda}(\mathbf{k},\mathbf{p})\Big[3\Phi_{\mathbf{p}}\Phi_{\mathbf{k}-\mathbf{p}} + \mathcal{H}^{-2}\Phi'_{\mathbf{p}}\Phi'_{\mathbf{k}-\mathbf{p}} + \mathcal{H}^{-1}\Phi'_{\mathbf{p}}\Phi_{\mathbf{k}-\mathbf{p}} + \mathcal{H}^{-1}\Phi_{\mathbf{p}}\Phi'_{\mathbf{k}-\mathbf{p}}\Big]$$



The projection:

$$\mathbf{e}^{\lambda}(\mathbf{k},\mathbf{p}) \equiv e_{lm}^{\lambda}(\mathbf{k})p_l p_m = \begin{cases} \frac{1}{\sqrt{2}}p^2(1-\mu^2)\cos 2\varphi, & \lambda = + \\ \frac{1}{\sqrt{2}}p^2(1-\mu^2)\sin 2\varphi, & \lambda = \times \end{cases}, \quad \mu \equiv \frac{\mathbf{k}\cdot\mathbf{p}}{kp} = \cos\theta$$

K. N. Ananda, C. Clarkson, and D. Wands, Phys. Rev.D75, 123518 (2007)

D. Baumann, P. J. Steinhardt, K. Takahashi, and K. Ichiki, Phys. Rev. D76, 084019 (2007)

- **The correlator of the GWs:**

$$\langle \hat{h}^{\lambda}_{\mathbf{k}}(\tau)\hat{h}^{s}_{\mathbf{k}'}(\tau)\rangle = \frac{1}{a^2(\tau)}\int^{\tau} \mathrm{d}\tau_1 \int^{\tau} \mathrm{d}\tau_2 g_k(\tau,\tau_1)g_k(\tau,\tau_2)a(\tau_1)a(\tau_2)\langle \hat{S}^{\lambda}_{\mathbf{k}}(\tau_1)\hat{S}^{s}_{\mathbf{k}'}(\tau_2)\rangle$$

The Green function:

$$g_k(\tau,\tau_1) = \frac{1}{k}\sin(k\tau - k\tau_1)\Theta(\tau-\tau_1)$$

Evolution of the Bardeen potential:

$$\Phi_{\mathbf{k}}(\tau) \equiv \frac{2}{3}T(k\tau)\zeta_{\mathbf{k}} \qquad T(z) = \frac{9}{z^2}\left[\frac{\sin(z/\sqrt{3})}{z/\sqrt{3}} - \cos(z/\sqrt{3})\right]$$

Four-point correlator （Wick’s theorem）：

$$\langle \hat{\zeta}_{\mathbf{p}}\hat{\zeta}_{\mathbf{k}-\mathbf{p}}\hat{\zeta}_{\mathbf{q}}\hat{\zeta}_{\mathbf{k}'-\mathbf{q}}\rangle = \langle \hat{\zeta}_{\mathbf{p}}\hat{\zeta}_{\mathbf{k}-\mathbf{p}}\rangle\langle \hat{\zeta}_{\mathbf{q}}\hat{\zeta}_{\mathbf{k}'-\mathbf{q}}\rangle + \langle \hat{\zeta}_{\mathbf{p}}\hat{\zeta}_{\mathbf{q}}\rangle\langle \hat{\zeta}_{\mathbf{k}-\mathbf{p}}\hat{\zeta}_{\mathbf{k}'-\mathbf{q}}\rangle + \langle \hat{\zeta}_{\mathbf{p}}\hat{\zeta}_{\mathbf{k}'-\mathbf{q}}\rangle\langle \hat{\zeta}_{\mathbf{k}-\mathbf{p}}\hat{\zeta}_{\mathbf{q}}\rangle$$

However, $\langle \hat{\zeta}_{\mathbf{p}}\hat{\zeta}_{\mathbf{k}-\mathbf{p}}\rangle = (2\pi)^3\frac{2\pi^2}{p^3}\delta^{(3)}(\mathbf{k})P_{\zeta}(p)$ vanishes when $\boldsymbol{k} \neq 0$

- **The power spectrum of the induced GWs:**

$$\mathcal{P}_h^{\rm RD}(k,\tau)=\int_0^\infty \mathrm{d}y\int_{|1-y|}^{1+y}\mathrm{d}x\left[\frac{4y^2-(1+y^2-x^2)^2}{4xy}\right]^2\mathcal{P}_\zeta(kx)\mathcal{P}_\zeta(ky)F_{\rm RD}(k,\tau,x,y)$$

where

$$F_{\rm RD}(k,\tau,x,y)=\frac{4}{81}\frac{1}{z^2}\left[\cos^2(z)\mathcal{I}_c^2+\sin^2(z)\mathcal{I}_s^2+\sin(2z)\mathcal{I}_c\mathcal{I}_s\right]\,.$$

and the functions $\mathcal{I}_c$ and $\mathcal{I}_s$ are given by

$$\mathcal{I}_c(x,y)=4\int_1^\infty \mathrm{d}z_1(-z_1\sin z_1)\Big\{2T(xz_1)T(yz_1)+\left[T(xz_1)+xz_1T'(xz_1)\right]\left[T(yz_1)+yz_1T'(yz_1)\right]\Big\}\,,$$

$$\mathcal{I}_s(x,y)=4\int_1^\infty \mathrm{d}z_1(z_1\cos z_1)\Big\{2T(xz_1)T(yz_1)+\left[T(xz_1)+xz_1T'(xz_1)\right]\left[T(yz_1)+yz_1T'(yz_1)\right]\Big\}\,,$$

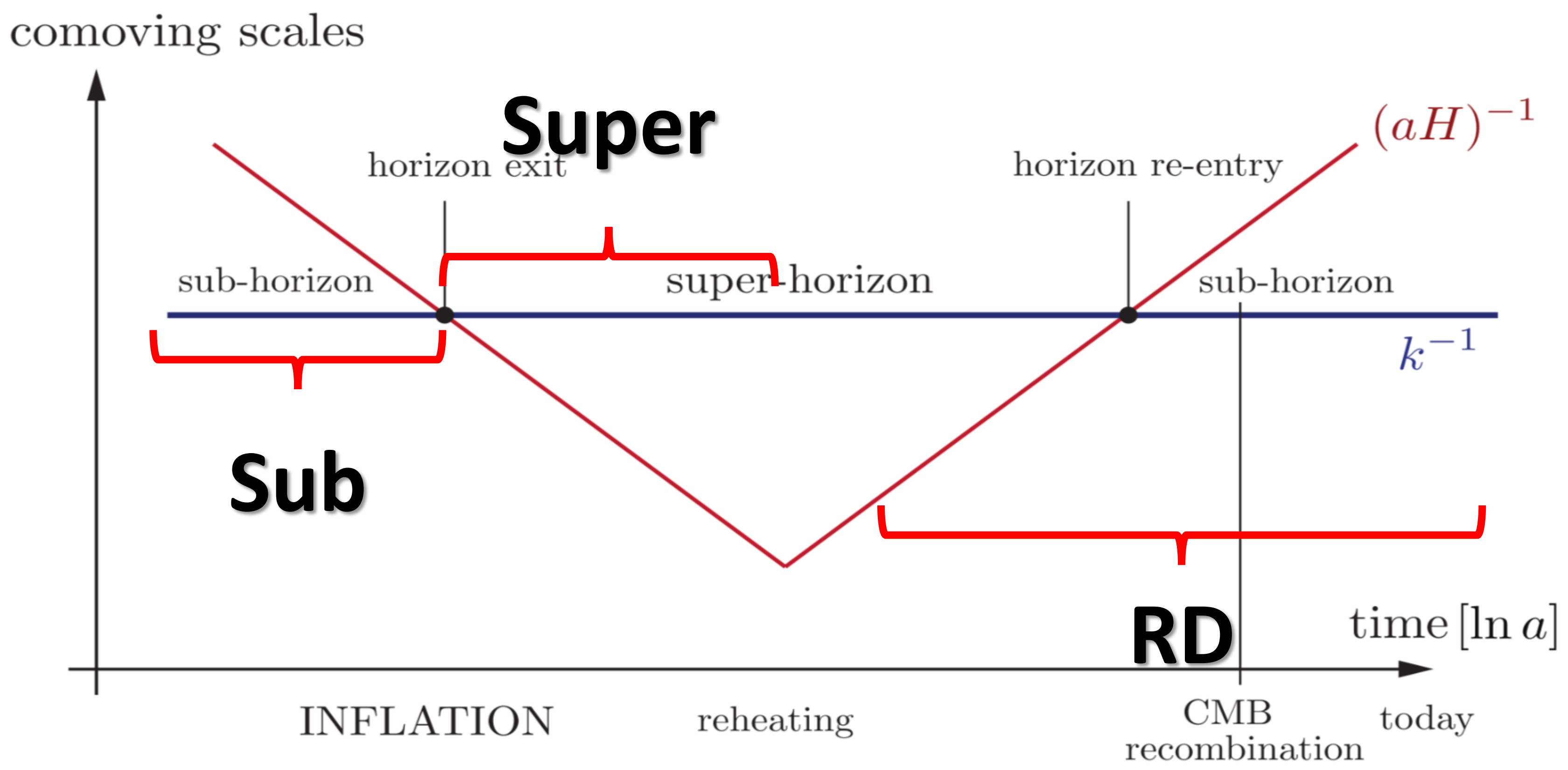
respectively, where $z_1=k\tau_1$. And we have introduced the variables $x=|\mathbf{k}-\mathbf{p}|/k$ and $y=p/k$.

For the power spectrum of the primordial curvature perturbation of any shapes, one could obtain the corresponding power spectrum for the induced GWs.

N. Bartolo, V. De Luca, G. Franciolini, M. Peloso, and A. Riotto, (2018), arXiv:1810.12218 [astro-ph.CO]

From the inflationary era to the radiation-dominated era:

In our recent work (1902.08187), we studied the possibilities of producing the induced GWs from inflation to radiation in the SSR mechanism.



Induced Gravitational Waves in SSR Mechanism (Super-Hubble Scale)

- **On the super-Hubble scales**

The perturbed inflation could provide the anisotropic stress which induces the GWs:

$$S_{\mathbf{k}}^{\lambda}(\tau) = 4\frac{c_s^2}{M_p^2}\int \frac{\mathrm{d}^3\mathbf{p}}{(2\pi)^3}\mathbf{e}^{\lambda}(\mathbf{k},\mathbf{p})\delta\phi_{\mathbf{p}}(\tau)\delta\phi_{\mathbf{k}-\mathbf{p}}(\tau)$$

The power spectrum for the GWs:

$$\langle \delta\hat{\phi}_{\mathbf{p}}\delta\hat{\phi}_{\mathbf{k}-\mathbf{p}}\delta\hat{\phi}_{\mathbf{q}}\delta\hat{\phi}_{\mathbf{k}'-\mathbf{q}}\rangle$$

$$\mathcal{P}_h^{\mathrm{Super}}(k,\tau) = \int_0^{\infty}\mathrm{d}y\int_{|1-y|}^{1+y}\mathrm{d}x\left[\frac{4y^2-(1+y^2-x^2)^2}{4xy}\right]^2\mathcal{P}_{\zeta}(kx)\mathcal{P}_{\zeta}(ky)F_{\mathrm{Inf}}(k,\tau,u)$$

Since the period of the sound speed oscillation c_s is larger than the period from the Hubble crossing to the end of inflation, and the amplitude of oscillation (i.e. ξ) is also small, hence the function $F_{\mathrm{Inf}}(k,\tau,u)$ is approximately

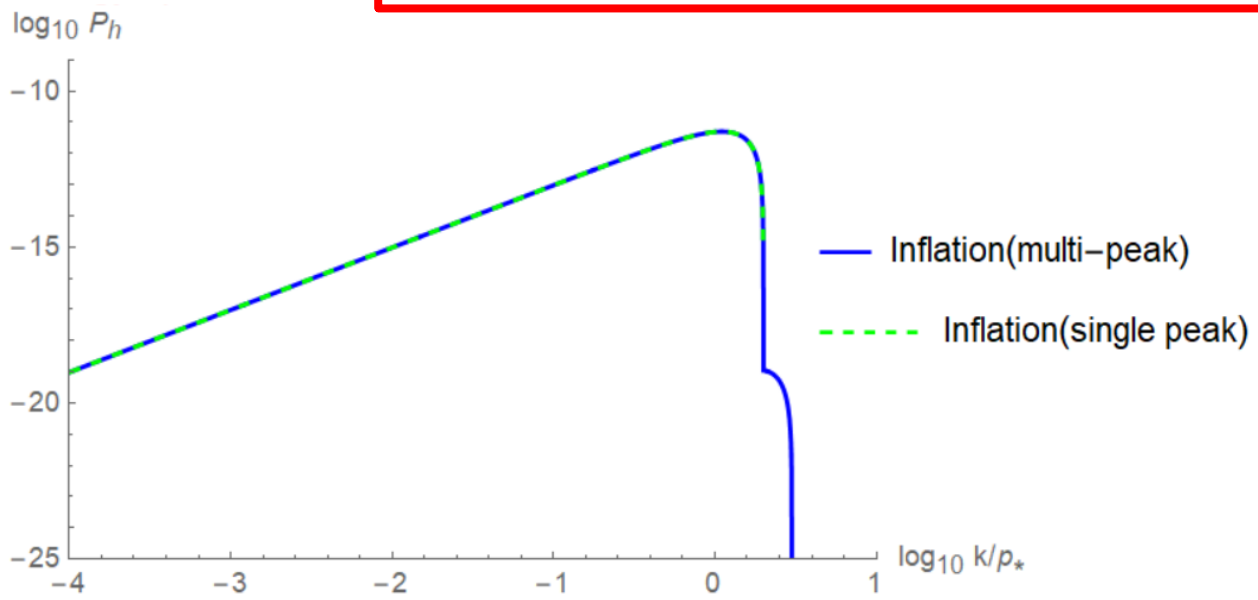
$$F_{\mathrm{Inf}}(k,\tau,u) \simeq 16\epsilon^2u^{-2}\Big[u+z\cos(z+u)-\sin(z+u)\Big]^2 ,$$

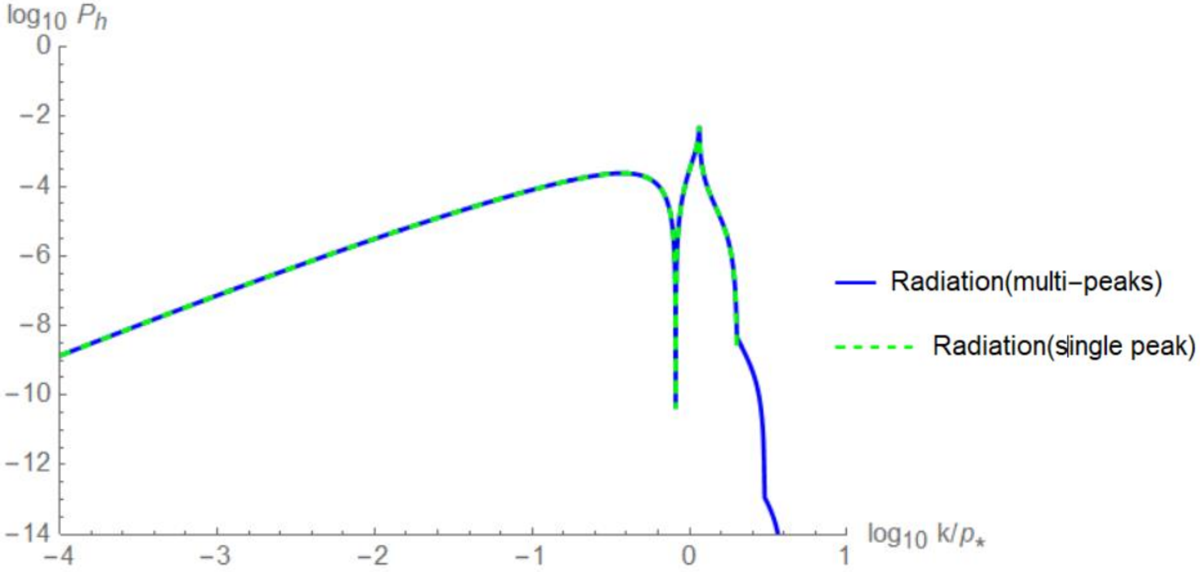
where $u = k/p_*$.

- **Comparison with the RD**

$$\mathcal{P}_h^{\mathrm{Super}}(k,\tau)=\int_0^\infty \mathrm{d}y\int_{|1-y|}^{1+y}\mathrm{d}x\left[\frac{4y^2-(1+y^2-x^2)^2}{4xy}\right]^2\mathcal{P}_\zeta(kx)\mathcal{P}_\zeta(ky)F_{\mathrm{Inf}}(k,\tau,u)$$

$$\mathcal{P}_h^{\mathrm{RD}}(k,\tau)=\int_0^\infty \mathrm{d}y\int_{|1-y|}^{1+y}\mathrm{d}x\left[\frac{4y^2-(1+y^2-x^2)^2}{4xy}\right]^2\mathcal{P}_\zeta(kx)\mathcal{P}_\zeta(ky)F_{\mathrm{RD}}(k,\tau,x,y)$$





Induced Gravitational Waves in SSR Mechanism (Sub-Hubble Scale)

- **On the sub-Hubble scale**

In principle, one could perform the same procedure as before:

$$\langle \delta\hat{\phi}_{\mathbf{p}}(\tau_1)\delta\hat{\phi}_{\mathbf{k}-\mathbf{p}}(\tau_1)\delta\hat{\phi}_{\mathbf{q}}(\tau_2)\delta\hat{\phi}_{\mathbf{k}'-\mathbf{q}}(\tau_2)\rangle$$

It's not easy to calculate this formula, e.g. the time ordering.

The power spectrum:

$$\langle 0|\hat{h}^{ij}(\tau,\mathbf{x})\hat{h}_{ij}(\tau,\mathbf{x})|0\rangle = \frac{1}{(2\pi)^3}\int d^3\mathbf{k}\sum_{\lambda=+,\times}|h_k^{\lambda}(\tau)|^2 \qquad \langle 0|\hat{h}^{s}_{\mathbf{k}}(\tau)\hat{h}^{\lambda}_{\mathbf{p}}(\tau)|0\rangle = \delta^{\lambda s}\delta^{(3)}(\mathbf{k}+\mathbf{p})\frac{2\pi^2}{k^3}P_h(k,\tau)$$

$$\mathcal{P}_h^{\mathrm{Sub}}(k,\tau_*) = \frac{4}{\pi^4 M_p^4}k^3\int_0^{\infty} dp\,p^6\int_0^{\pi} d\theta\sin^5\theta$$

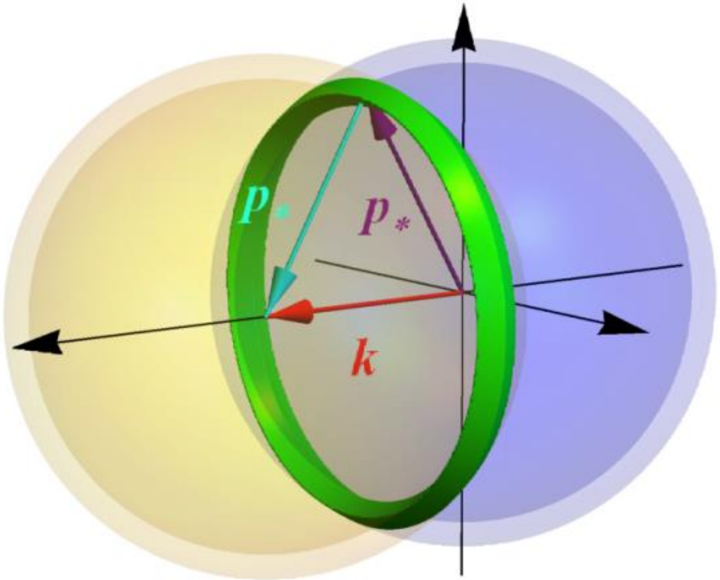
Phase integral

$$\times\left|\int_{\tau_i}^{\tau_*} d\tau_1 c_s^2(\tau_1)g_k(\tau_*,\tau_1)\delta\phi_p(\tau_1)\delta\phi_{|\mathbf{k}-\mathbf{p}|}(\tau_1)\right|^2$$

Time integral

M. Biagetti, M. Fasiello, and A. Riotto, Phys. Rev. D88, 103518 (2013), arXiv:1305.7241 [astro-ph.CO].

- **The phase integral: the thin-ring approximation**



The major contribution to $\mathcal{P}_h$:

The sub-Hubble modes of $\delta\widehat{\phi_p}$ in the neighborhood of the characteristic scale p_* can be exponentially amplified.

The thing ring:

The effective space for the phase integral is thus the subspace of the overlapping region.

Phase integral

$$\Delta\Pi = \int dpp^2 d\cos\theta d\varphi = \begin{cases} 2\pi\xi^2 p_*^4/k, & k > \frac{\xi}{2}p_* \\ 4\pi\xi p_*^3, & k < \frac{\xi}{2}p_* \end{cases}$$

- **The time integral: a numerical way**

Complicated solution: $$v_{p_*}(\tau) = \frac{H\tau}{\sqrt{2p_*}} e^{iv} \frac{S(s)\left(iC(v) - C'(v)\right) + C(s)\left(-iS(v) + S'(v)\right)}{-S(v)C'(v) + C(v)S'(v)}$$

Time integral:

$$\int_{\tau_i}^{\tau_*} d\tau_1 c_s^2(\tau_1) g_k(\tau_*, \tau_1) \delta\phi_p(\tau_1) \delta\phi_{|\mathbf{k}-\mathbf{p}|}(\tau_1) = \frac{e^{iu+2iv}}{4p_*^5 u^3} H^2 \int_1^v ds \left[1 - 2\xi(1 - \cos(2s))\right] M^2(s, v) \times e^{ius}\left[e^{-2iu}(1+iu)(-i-us) + e^{-2ius}(1-iu)(i-us)\right]$$

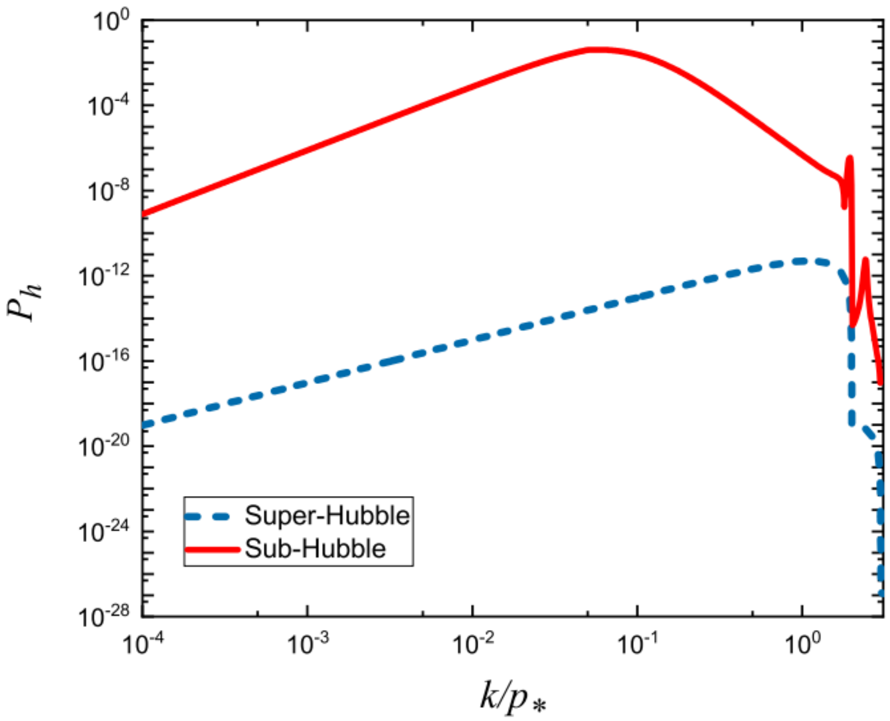
The power spectrum:

$$\mathcal{P}_h^{\rm Sub}(k, \tau_*) = \begin{cases} 16\xi^2\epsilon^2 A_s^2 \left(1 - \frac{u^2}{4}\right)^2 \frac{1}{u^4} |\mathcal{I}(u, v)|^2 \,, & u > \frac{\xi}{2}, \\ 32\xi\epsilon^2 A_s^2 \left(1 - \frac{u^2}{4}\right)^2 \frac{1}{u^3} |\mathcal{I}(u, v)|^2 \,, & u < \frac{\xi}{2}, \end{cases}$$

$$\mathcal{I}(u, v) = \int_1^v ds \left[1 - 2\xi(1 - \cos(2s))\right] e^{ius} \times \left[e^{-2iu}(1+iu)(-i-us) + e^{-2ius}(1-iu)(i-us)\right] \times \left[\frac{S(s)\left(iC(v) - C'(v)\right) + C(s)\left(-iS(v) + S'(v)\right)}{-S(v)C'(v) + C(v)S'(v)}\right]^2 .$$

Induced Gravitational Waves in SSR Mechanism (Sub-Hubble Scale)

➢ The contributions from the inflationary era



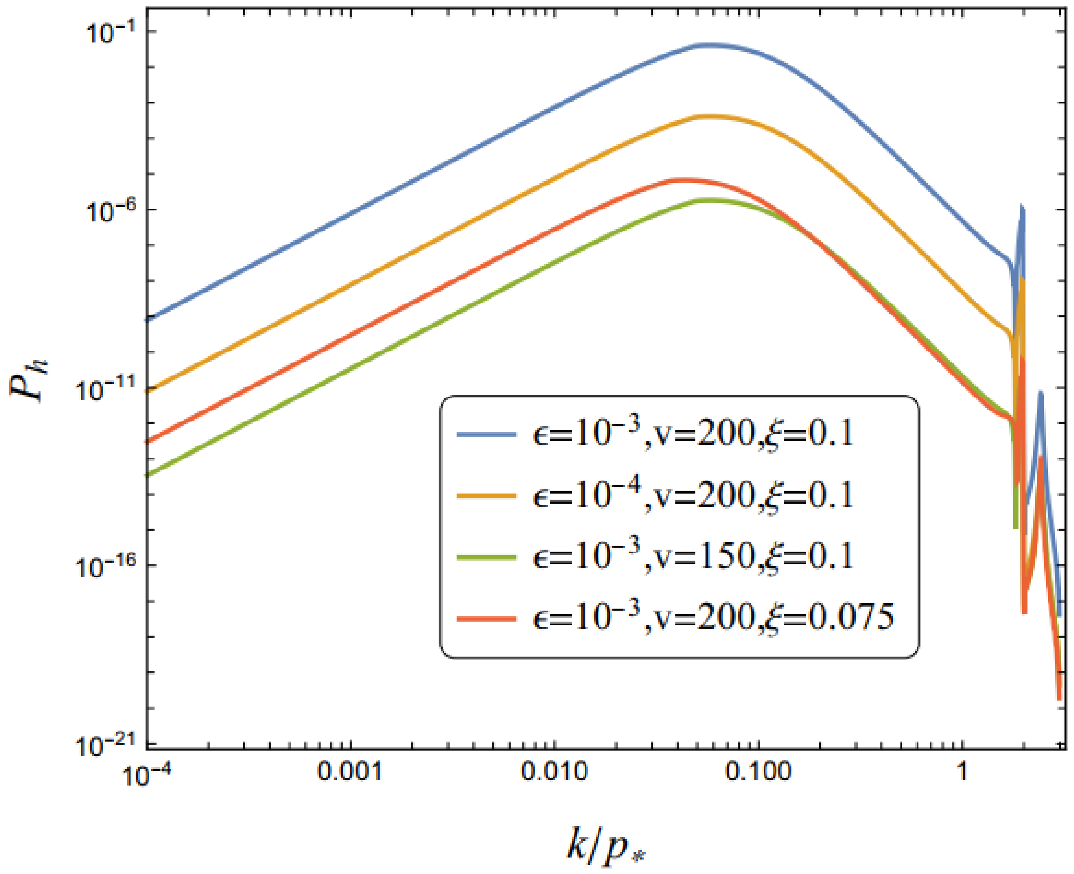
The values of parameters we used as follows: $\epsilon = 10^{-3}$, $v = 200$, $\xi = 0.1$ and $n_s = 0.968$.

The main contribution from the sub-Hubble scale:

(1) The expansion of the Universe from the beginning of the sound speed oscillation to the Hubble crossing of the p_* mode: $(\tau_i - \tau_*)/\tau_* \simeq e^{\Delta N}$, where ΔN is the e-folding number for this period of inflation. The GWs induced by the super-Hubble modes are actually contributed from the integral $\int_{\tau_*}^{\tau_{\rm end}}$ in $\mathcal{P}_h$, i.e., from the Hubble crossing of the p_* mode to the end of inflation $\tau_{\rm end}$: $(\tau_* - \tau_{\rm end})/\tau_* \simeq 1$. In the SSR mechanism, $e^{\Delta N} \sim \mathcal{O}(100)$ is quite larger than 1.

(2) During the late stage of the resonance, the mode function $\delta\phi_{p_*}(\tau)$ oscillates in a trigonometric way, giving rise to a non-vanishing central value of $\delta\phi_{p_*}^2$ which accumulates in the time integral in $\mathcal{P}_h$.

➢ Dependence on the parameters



The power spectra are diminished by decreasing any of them while the shapes of the curves remain barely changed.

$v = -p_* \tau_i$ controls the duration of resonance; ξ controls the rate of amplification; ϵ appears in the overall normalization.

Induced Gravitational Waves in SSR Mechanism

➤ **The energy spectra observed at the present**

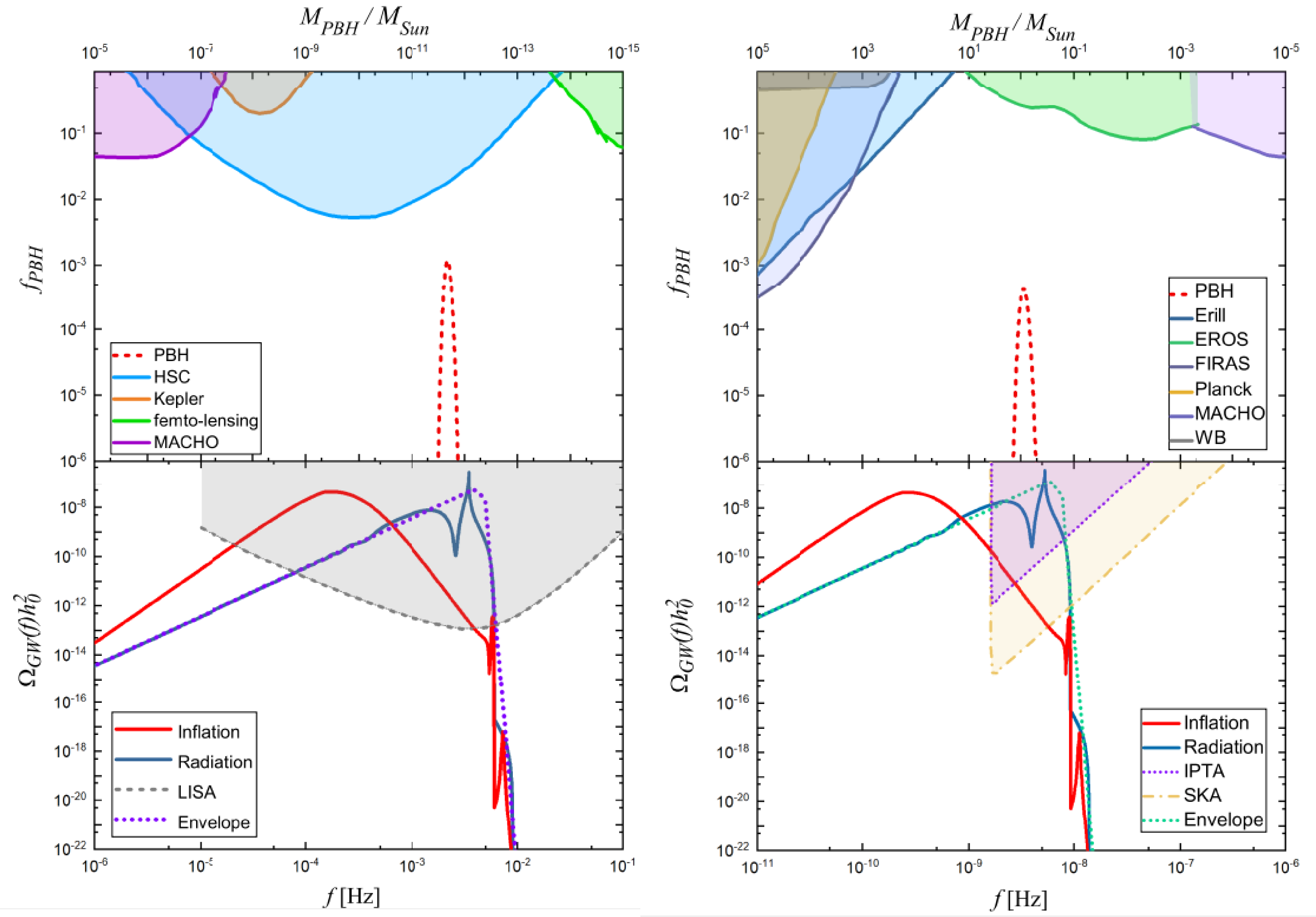
$$\Omega_{\rm GW}(\tau_0,k)\simeq\begin{cases}\Omega_{r,0}\Omega_{\rm GW}(\tau_f,k) & \text{RD}\\ 1.08\times10^{-6}\mathcal{P}_h^{\rm Inf}(k,\tau_{\rm end}) & \text{Inflation}\end{cases}$$

➤ **The parametrization**

$$\Omega_{\rm GW}(f)h_0^2\sim\begin{cases}A\left(\frac{f}{f_c}\right)^{\alpha}, & f<f_c,\\ A\left(\frac{f}{f_c}\right)^{\beta}, & f>f_c\ ,\end{cases}$$

$$A\simeq4.31\times10^{-7},\quad \alpha\simeq3.0,\quad \beta\simeq-5.4,\quad f_c=0.08f_*\ . \qquad \text{Inflation}$$

$$A\simeq8.41\times10^{-8}\ \text{(LISA)},\quad A\simeq2.05\times10^{-7}\ \text{(SKA\&IPTA)},\quad \alpha\simeq2.0,\quad \beta\simeq-50.4,\quad f_c=1.6f_*\ . \qquad \text{RD}$$



Summary

- We studied the induced GWs in the SSR mechanism, which originate from both the inflationary era and the radiation-dominated phase.

- The signal of the induced GWs by the sub-Hubble modes is much larger than the GWs induced by the sub-Hubble modes.

- Summing up all contributions, the energy spectra of the induced GWs display a unique double-peak pattern, which is potentially detected by forthcoming GW experiments, like LISA, SKA and IPTA. The GW observation could provide constraints on the SSR mechanism.